

**Aggregate Planning in  
Make-to-Order Environments  
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## **Abstract**

An aggregate plan for a make-to-order steel fabrication plant for a set of product archetypes is developed. An aggregate linear programming model is used to develop the aggregate plan, taking into account the difficulties that arise in planning in a make-to-order environment. A description of the facility modeled, the aggregate plan, and implications are discussed. Significantly cost savings for the implementation of the aggregate plan results. Further analysis will be conducted to then create a production schedule for the facility.

**Key Words:** Aggregate Planning, Linear Programming, Make-to-Order

## **1. Introduction**

Aggregate planning in make-to-order environments is challenging due to the inability to store large amounts of inventory to buffer against large changes in demand. It is also more challenging due to the larger number of different products that can be produced. However, the need to plan for production in these environments cannot be overlooked. Just as with make-to-stock production, the need to keep inventory levels low, meet due date requirements, and efficiently operate the facility are paramount to a company's success and competitiveness in their respective industry. The steel fabrication industry is no exception, especially with the current need for steel fabrication companies to produce fabricated products quickly, at a reasonable cost, and within a global environment.

Our approach, due to simplicity, ease in understandability of management, and the little marginal gain in accuracy from more advanced methods, is to use a linear programming approach to develop the aggregate production plan for a steel fabrication plant located in the Midwest. A linear programming model that minimizes total costs for an aggregate production plan is developed. This model requires forecasts of aggregate monthly demand, and in turn yields a production plan for each month over a twelve month rolling horizon. Other methods described in the literature that have been used to develop aggregate plans consist of nonlinear integer programming techniques (Rajagopalan, 2002), heuristics or algorithmic techniques (Dejonckherre et. al, 2003 and Chang et. al, 2003), goal programming techniques (Leung et. al, 2003) and fuzzy logic (Fung, et. al, 2003). In addition, Rao et. al (2004) utilize a two-stage integer stochastic program in which they determine the products to produce and how much to produce, similar outputs of an aggregate plan.

One item inherent to aggregate planning is that certain limitations on production exist. These limitations can include the number of employees, the capacities of the resources, the amount of storage space for work-in-process (WIP) inventory or finished product inventory, demand placed on the products, hours within a given work week, and even other managerial limitations. Typically, a linear programming model incorporating such constraints is used in the aggregate plan to evaluate a cost minimizing objective function. The production plan approach and its benefits are well established in the literature by Bitran and Hax (1977), Leong, Oliff, and Markland (1989), Bowers and Jarvis (1992), Qui, Fredendall, and Zhu (2001), Rogers, Plante, Wong, and Evans (1991) and others. Further, and more recently, research has been conducted on production planning in hybrid make-to-stock and make-to-order environments. Rajagopalan (2002) has developed a non-linear integer program to aid in the decision of which items to make-to-stock and which items to make-to-order for a large consumer product manufacturer and Chang, Pai, Tuan, Wang, and Li (2003) have developed a hybrid model for aggregate planning in the semiconductor industry. In addition, Tsubone, Ishikawa, and Yasamoto (2002) have designed a production planning system in a hybrid environment using the hierarchical production planning approach.

## **2. General background**

### **2.1 Description of the fabrication plant**

The steel fabrication facility used to model the aggregate plan is a century-old plant located in the industrial Midwest. Fourteen machines are used to fabricate thousands of different end products, from which twelve product archetypes have been identified (see Rogers et. al, 1991). Each product is classified by class, as industrial or commercial; by type, as angle, beam,

or plate; and by weight, as light or heavy. Production sequences, set-up times and processing times vary among these archetypes.

The plant consists of two large hanger type buildings, similar to those that house airplanes in the military. The first building houses the majority of the newer fabrication machines, with an old 'Iron Worker' dead set in the middle. Special fabrication occurs in this area. The second building consists of four rooms, the first of which consists of only a few machines and a fabrication center. In the second room stands a storage area where shipment, receipt, and painting occur. Adjacent to the storage area is the third room that contains an active burning machine and several cutting machines. An alley leads into the final room that consists of angle and plate roll machines, a programmable press break, light and heavy shears, and two fabrication areas. This area is generally used for large fabrication of plates and angles, with the programmable break machine the 'envy' of the plant.

The plant operates on a five-day workweek with one eight-hour shift per day. Overtime can be scheduled on Saturday exclusively. A corporate directive constrains the workforce to remain between twenty-five and thirty employees, inclusively, at all times. Adjustment of the workforce within these limits occurs due to seasonal changes in demand. Past experience has taught management to prohibit backordering; in the recent past, the sales volume of backorders had reached \$600,000.

Management follows a strictly make-to-order environment, scheduling production only when an order is received. This policy obviates the need to prioritize between make-to-stock and make-to-order products. The challenge of forecasting demand for products in this environment is discussed in section 3.

## **2.2 Data collection**

The data for this study was mainly gathered through lengthy discussions with the President of the company. Accounting information was provided for the previous year so that forecasts could be generated. The set-up and machine run times were not documented so the expertise of the President, who has been in this industry for more than 35 years, was utilized to estimate run times and set-up times for the machines. Hiring and firing costs were partly determined by the accounting information and by discussions with the President. Hourly wages were determined from the accounting information in which an average of all the currently employed workers was determined. Overtime was based on one and one half times the average regular wage rate. Finally, the President of the firm estimated the average worker utilization. The average worker utilization is an important parameter because, occasionally, workers are required simultaneously in production. The data used in the aggregate model can be found in tables A-1 and A-2 of Appendix A.

### **3. Forecasting models**

Forecasting techniques play a significant role in aggregate planning. Consistent forecasts for all product classes in the aggregate plan are obtained by a technique due to Leong et al (1989). Forecasts are handed down from the corporate office, and specific class demands are determined as weighted amounts of the overall archetype demanded. Thus, our forecast amounts for a particular month for a given archetype,  $F_m$ , are determined as follows:

$$F_m = W*(FD), \quad (3-1)$$

where  $W$  is the weight determined by the monthly demand (monthly demand divided by the number of the product archetype produced in a month and converted to a percentage) and  $FD$  is the fabricated demand which constitutes eighty percent of the monthly demand (twenty percent

of the demanded material is bought and sold as is, therefore, no fabrication or production processing is required).

#### **4. The Aggregate plan**

With forecasted monthly demand at hand, our next goal is to allocate production capacity among six product archetypes consisting of two classes (commercial and industrial) and three types (angles, plates, and beams). Differences in the weight categories of light and heavy are only relevant in more precise planning. As customary we look at the overall production plan on a twelve-month rolling horizon. Bitran and Hax's (1977) design of production systems, Bitran, Haas and Hax's (1981, 1982) single stage production system and two-stage production system, Hax and Meal's (1975) production planning and scheduling model, Bowers and Jarvis' model (1992), and Techawiboonwong and Yenradee's aggregate planning model (2003) provide the basis for the model developed.

Customarily, the master production schedule is developed for selected items (see Bitran et al., 1981). However, it is worth noting that in this highly customized, make-to-order environment with thousands of different possible end products (some of which have never been produced before) the master production schedule is more easily developed for product archetypes (see Bitran et al., 1982). Even though each specific item in the archetype is manufactured differently, the clustering into product archetypes minimizes this difference and, therefore, has little effect on the efficiency of the master production schedule.

The aggregate linear programming model seeks to minimize total costs, consisting of inventory holding, regular-time and overtime, and hiring and firing (Bowers and Jarvis, 1992). The decision variables are the amounts of each product class to be produced, the inventory at the

end of each month, the total number of workers employed in a given month, regular and overtime labor in hours, and the number of hires and fires in a given month.

#### 4.1 The linear programming model

##### Decision Variables:

$X_{ijm}$  = amount of product, class  $i$ , type  $j$ , produced in month  $m$ .

$I_{ijm}$  = amount of inventory of product, class  $i$ , type  $j$ , at the end of month  $m$ .

$W_m$  = total number of workers employed in month  $m$ .

$R_m$  = number of regular man-hours used in month  $m$

$\theta_m$  = number of overtime man-hours used in month  $m$ .

$A_m$  = number of employees hired in month  $m$ .

$F_m$  = number of employees laid off in month  $m$ .

##### Parameters:

$h_{ij}$  = average holding cost for product of class  $i$ , type  $j$  per month.

$d_{ij}$  = forecast demand for product of class  $i$ , type  $j$ .

$p_{ij}$  = production rate for product of class  $i$ , type  $j$ .

$v_{ij}$  = avg. size ('footprint', sq. ft.) of product of class  $i$ , type  $j$ .

$b_{ij}$  = backorder cost for product of class  $i$ , type  $j$ .

$D_m$  = total unit demand in month  $m$ .

$\pi$  = worker productivity rate (units per month).

$rt$  = total number of available regular-time hours per month

$ot$  = total number of available overtime hours per month

$c$  = monthly production capacity of entire plant.

$s$  = storage capacity of entire plant.

$r$  = regular hourly wage rate.

$\omega$  = overtime hourly wage rate.

$f$  = cost to fire one employee.

$\alpha$  = cost to hire one employee.

### Index Sets:

product class  $i \in \{1 \text{ (commercial)}, 2 \text{ (industrial)}\}$ .

product type  $j \in \{1 \text{ (angle)}, 2 \text{ (beam)}, 3 \text{ (plate)}\}$ .

month  $m \in \{1, 2, \dots, 12\}$ .

$$\text{Minimize } \sum_i \sum_j h_{ij} \sum_m I_{ijm} + \sum_m [(rR_m + \omega\theta_m) + (fF_m + aA_m)] \quad (1)$$

s.t.

$$I_{ij(m-1)} + X_{ijm} - I_{ijm} \geq d_{ij}, \quad \forall \text{ class } i, \text{ type } j, \text{ and month } m; \quad (2)$$

$$\sum_i \sum_j \frac{X_{ijm}}{P_{ij}} \leq c, \quad \forall \text{ month } m; \quad (3)$$

$$\pi W_m \geq D_m, \quad \forall \text{ month } m. \quad (4)$$

$$\sum_i \sum_j (v_{ij} I_{ijm} + v_{ij} X_{ijm}) \leq s, \quad \forall \text{ month } m; \quad (5)$$

$$R_m \leq rt, \quad \forall \text{ month } m; \quad (6)$$

$$\theta_m \leq ot, \quad \forall \text{ month } m; \quad (7)$$

$$W_m - W_{m-1} = A_m - F_m, \quad \forall \text{ month } m; \quad (8)$$

$$ot + rt \leq 24(W_m) \quad (9)$$

$$25 \leq W_m \leq 30; \quad \forall \text{ month } m; \quad (10)$$

$$X_{ijm}, I_{ijm}, W_m, R_m, \theta_m, A_m, F_m \geq 0, \quad \forall \text{ class } i, \text{ type } j, \text{ and month } m. \quad (11)$$

The constraints enforce material balance of demand (2), total productive capacity (3), demand for labor (4), the physical storage limit (5), regular (6) and overtime (7) labor time availability, workforce smoothing (8), hours and workforce level (9), and workforce limits (10).

LINDO software was used to solve the aggregate linear program (ALP) to optimality. The optimal solution maintained the workforce at the lower limit of 25 throughout the 12-month horizon, and yielded an objective value of \$545,231.25. The months of June-October required overtime due to the production capacity available during regular time. The entire optimal solution is reported in table A-3 of Appendix A. The workforce restrictions (9) were then removed and a modified ALP was solved, yielding the workforce levels indicated in figure 2, with an optimal objective value of \$310,627.30 (see table A-4 of Appendix A). This modified ALP was solved to demonstrate to the President the effects of his self-imposed labor constraints. It is not unexpected that the master production schedule is the same except the labor requirement is much smaller. This is quite plausible since this environment is more machine than labor intensive.

## **5. Implications**

The firm studied should immediately realize an approximate savings of \$234,603.95 in labor costs for the year due to abandoning its current labor policy and following the chase strategy outlined in the aggregate model. However, the authors are fully aware of the difficulties involved with following a chase strategy due to the number of layoffs involved. Another solution that we put forth is to reduce the entire labor force from a lower bound of 25 to a lower bound of 17 (the labor required in the highest volume month). This would then result in a yearly savings of approximately \$174,720. In addition to these savings, the estimated \$600,000 backlog can be eliminated creating an additional savings from the cost of this backlog (not

readily quantifiable). Finally, further cost savings are realized by the long-run effects of increased service (in terms of lead-time reduction), increased cash flow due to lower work-in-process inventory, customer retention, and customer loyalty.

The next step of this research is the development of a second and third tier solution so that the aggregate plan may be used to develop a full-fledged production plan for the year, broken down by month. At this time, it is proposed that a non-linear program be used to disaggregate the aggregate plan by minimizing set up costs. Further, it is proposed that a sequencing heuristic be used to then sequence specific jobs through specific flows through the facility. It is hypothesized that the addition of these two tiers will further decrease costs and improve customer service.

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### Appendix A. Aggregate Level Data and Optimal Production Plan.

	<b>CA: Commercial Angles</b>	<b>CB: Commercial Beams</b>	<b>CP: Commercial Plates</b>	<b>IA: Industrial Angles</b>	<b>IB: Industrial Beams</b>	<b>IP: Industrial Plates</b>
Holding Costs (\$/mo.)	86.67	650.00	130.00	46.67	93.33	326.67
Production Rate (units/mo.)	100	75	46	100	75	46
Average Footprint (sq. ft.)	2	8	16	4	12	36

Table A-1: Aggregate Planning Data, Archetype Dependent

Regular time labor available per month:	15,600 hrs.
Regular time wage rate:	\$10.50/hr.
Overtime labor available per month:	3120 hrs.
Overtime wage rate:	\$15.75/hr.
Production capacity (machine dependent inclusive of Saturday):	12,500 units/mo.
Storage capacity:	17,500 sq. ft.
Hiring cost:	\$1000/worker
Firing cost:	\$860/worker

Table A-2: Aggregate Planning Data, Non-Archetype Dependent.

	<b>Jan</b>	<b>Feb</b>	<b>Mar</b>	<b>Apr</b>	<b>May</b>	<b>Jun</b>	<b>Jul</b>	<b>Aug</b>	<b>Sep</b>	<b>Oct</b>	<b>Nov</b>	<b>Dec</b>
CA prod	10	12	14	14	14	15	16	16	17	16	14	14
CB prod	74	90	106	106	106	114	122	122	130	122	106	106
CP prod	14	18	21	21	23	25	25	25	26	25	21	21
IA prod	5	7	8	8	8	8	9	9	10	9	8	8
IB prod	10	14	16	16	16	17	19	19	21	19	16	16
IP prod	37	43	52	52	52	57	59	59	62	59	52	52
Workers	25	25	25	25	25	25	25	25	25	25	25	25
Reg Hrs	4325	4325	4325	4325	4325	4325	4325	4325	4325	4325	4325	4325
OT Hrs	0	0	0	0	0	1.29	3.57	3.57	5.86	3.57	0	0

Table A-3: Aggregate Model with Employee Restrictions. The optimal solution has an optimal objective value of 545,231.25 and includes no hires, fires, nor items in inventory.

	<b>Jan</b>	<b>Feb</b>	<b>Mar</b>	<b>Apr</b>	<b>May</b>	<b>Jun</b>	<b>Jul</b>	<b>Aug</b>	<b>Sep</b>	<b>Oct</b>	<b>Nov</b>	<b>Dec</b>
CA prod	10	12	14	14	14	15	16	16	17	16	14	14
CB prod	74	90	106	106	106	114	122	122	130	122	106	106
CP prod	14	18	21	21	23	25	25	25	26	25	21	21
IA prod	5	7	8	8	8	8	9	9	10	9	8	8
IB prod	10	14	16	16	16	17	19	19	21	19	16	16
IP prod	37	43	52	52	52	57	59	59	62	59	52	52
Workers	8	11	13	13	13	15	16	16	17	16	13	13
Change by mo.	0	+3	+2	0	0	+2	+1	0	+1	-1	-3	0
Reg Hrs	1384	1903	2249	2249	2249	2595	2760	2760	2941	2768	2249	2249
OT Hrs	0	0	0	0	0	1.29	3.57	3.57	5.86	3.57	0	0

Table A-4: Aggregate Model without Employee Restrictions. The optimal solution has an objective value of \$289,440.80 and includes no items in inventory.